Propulsion by Undulating Plates

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The usefulness of undulating plates for underwater propulsion was investigated in a brief experimental program and by reviewing the relevant literature. A watercraft, consisting of two aircraft drop tanks, which act as floats and are interconnected in catamaran fashion by a steel framework, was built especially for the purpose. The undulating plate of small aspect ratio is mounted horizontally in the centerline about 2 ft below water level. The plate is hinged near the leading edge in a support frame and is excited harmonically at one-third of the chord. Ten different plates, all about 1 × 3 ft, were tested. The maximum speed obtained was 5.1 fps. Maximum propulsive efficiencies obtained in the tests were only 16%. However, theoretical work indicates that extremely high efficiencies may occur in certain designs. If the efficiencies can be improved, applications are likely because of one or more of the following advantages: no shaft-sealing problems, low acoustic noise level, small idling drag, good thrust control, or safe environment for swimmers. However, speeds will probably remain below 15 knots. A systematic parameter study is required to determine the highly efficient configurations.

Nomenclature

c = chord length, in.
c_T = thrust coefficient
f = frequency, cps
j = advance ratio of propeller
k = reduced frequency, $\omega c/2U$
L = wavelength of flexural wave, in.
N = nose length, in.
n = wave number, $(\pi c/L)$
P = total power transmitted to plate, in.-lb-sec^{-1}
T = total thrust, lb
T_l = tail length, in.
U = freestream velocity, in.-sec^{-1}
V = phase velocity of flexural wave, in.-sec^{-1}
w = width of plate, in.
δ = plate thickness, in.
δ_l = tail thickness, in.
η = propulsive efficiency, TU/P
ρ = fluid density, lb-sec^2-in.$^{-4}$
ω = circular frequency, rad-sec^{-1}

Introduction

A PROBLEM that has been regarded as serious in some quarters of underwater technology is how to seal a rotating propeller shaft effectively under the high external pressures occurring at great depth. Getting rid of the rotating shaft and discarding the propeller in favor of undulating plates is a radical solution that no designer would be ready to propose at present. Nevertheless, if the problem is as serious as was suggested, an investigation of the insufficiently understood propulsive characteristics of undulating plates seems justified, resulting in a better appreciation of their potential.

The idea that led to the present investigation is essentially that shaft seals can be avoided in a propulsion system that consists of an oscillating member, supported at the point where it pierces the hull. At that point a flexible but permanent seal can be used. The power is transmitted by the oscillating member in the form of vibration. This can be generated at the end inside the hull by a mechanical oscillator, e.g., an eccentric rotating mass. The power is then transmitted to an undulating plate in the water which converts the vibratory energy into thrust.

The merit of this idea clearly depends on whether the possible solution of the shaft-sealing problem is worth the sacrifice of the propeller. In order to evaluate the extent of this sacrifice, it is necessary to know the characteristics of the undulating plate. The use of oscillating plates for underwater propulsion has been known for a long time, but its limitations and possibilities are not well known. It was the purpose of this investigation to define the problems involved in the determination of the thrust generated by undulating plates, to develop some insight into the characteristics of propulsion by means of undulating plates, and, if possible, to come to a conclusion with respect to practical applications.

In the following sections the information available in the literature will be reviewed, consisting partly of comments and observations on fish-propulsion and partly of theoretical investigations of unsteady hydrodynamics. The limited understanding derived from the literature leads to the definition of the problem area and the formulation of the required theoretical analysis. Next the experiments that have been performed are described, and finally some conclusions are drawn concerning possible future applications.

Fish-Propulsion Studies

The behavior of undulating flat plates is quite similar to that of fish. There is a substantial literature on the swimming of marine animals, containing mostly observations and discussions but also actual measurements of the performance of various animals. Although a detailed review of this literature would be out of place here, it seems worthwhile to summarize the main conclusions that are of interest in the present problem.

It has been shown that in some swimming animals there is a large discrepancy between the power expended in swimming and the power available on the basis of muscle weight. Some fish seem to do a factor of 5 or 10 better than can be expected, even assuming laminar flow over a large part of the body. It is suggested that the answer is a remarkable sensitivity to flow disturbances and local control to take advantage of these. In engineering terms this implies a large number of sensors and a variable excitation capability, both well distributed over the body length. The better swimmers among the marine animals usually have slender bodies, fairly heavy and stiff over the front half and lighter and more flexible toward the tail, which often has a relatively large span. Their

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performance suggests high propulsive efficiencies. The motion of the body can be interpreted as a wave traveling aft with increasing amplitude and a phase velocity greater than the freestream velocity. The wavelength is generally of the order of the body length, larger at the front and smaller at the tail, and the reduced frequency is often about 5.

This reduced frequency is defined as $k = \omega c/2U$, in which ω is the frequency of excitation in radians per second, c is the chord length, and U is the freestream velocity. Introducing the frequency f in cycles per second, it follows from $k \simeq 5$ that $f \simeq 5U/\pi c$. The phase velocity V of the wave traveling aft along the body can be expressed in the wavelength L as $V = f \cdot L \simeq 5\, UL/\pi c$. Hence, if the wavelength is of the order of the body length, as it seems to be over the forward half of the body, the phase velocity is larger than the freestream velocity. On the other hand, if it is smaller, say of the order of $(\pi c/5)$, as it is near the tail, the phase velocity is approximately equal to the freestream velocity. It is clear upon reflection that the thrust is generated only if the phase velocity is in excess of the freestream velocity. It appears, therefore, that the variation of the wavelength over the body is significant for the distribution of the thrust.

Since the instantaneous local thrust is the product of the local pressure and the local slope of the deflection curve, other variables of interest, besides the frequency and the wavelength, are the pressure distribution and the deflection amplitudes. All of these variables are, of course, coupled, and only a complete dynamic analysis can uncover their relationship.

The propulsive efficiency can be defined as $\eta = TU/P$, in which T represents the total thrust and P the total power.

In an interesting and useful study, in which he explains the essential characteristics of fish propulsion on the basis of hydrodynamic theory, Lighthill[1] comes to the conclusion that high propulsive efficiencies will occur if the phase velocity of the traveling wave is about 25% greater than the freestream velocity. Furthermore, the shape of the wave must be such that the amplitude increases from zero at the front to a maximum value at the tail and such that there is always a positive as well as a negative phase, in order to balance the forces and minimize angular recoil. The presence of the latter, known in dynamic analyses as rigid-body pitching motion, appears to be detrimental to the efficiency, in contrast to the presence of rigid-body transverse motion, which does not affect the efficiency. The span of the tail must also exceed a critical value, and the slope of the wave envelope at the tail must be zero for high efficiencies.

These conclusions are valid for free (unsupported) slender bodies. It should be noted that from the viewpoint of the dynamic analyst these bodies have a negligible stiffness and inertias of the same order as the virtual mass of the surrounding water. Therefore, the external pressure distribution is balanced to a large extent directly by the internal muscle forces. This seems to be a prerequisite for the high degree of control, which, together with the fine sensitivity to pressure fluctuations, leads to highly efficient swimming. As far as the internal forces are concerned, it is obvious that the body must have the capability of transmitting thrust and shear forces in order to maintain equilibrium with the external forces. Since the thrust forces are small of the second-order and the body usually is not capable of generating shear forces, it is clear that the body can actively operate upon the surrounding fluid only by generating local bending moments. In the case of a three-dimensional wave motion, bending moments would occur in two perpendicular axial planes. The integral of all internal bending moments over the body is practically equal and opposite to the total moment derived from the external pressures, the difference being only the moment necessary to overcome the inertia of the body in pitch, and this motion is small.

The preceding considerations serve to define the present problem. If flat plates are to be used in a propulsion system to generate thrust, it is likely that they will be excited and supported in a simple and practical way, so that the particular type of deflections and waves can occur which result in high propulsive efficiencies. The problem then naturally reduces to the determination of the influence of the design parameters, e.g., plate dimensions, location and nature of support, location of excitation point, frequency, etc., upon the propulsive characteristics. If this influence is known, an optimum set of dimensions can be selected for a particular purpose, and the propulsive characteristics of the resulting design can be evaluated.

The configuration that will be considered in particular from here on consists of a flat rectangular plate with varying thickness and small aspect ratio. This plate is supported at or near the leading edge, where it is free to rotate around an axis parallel to the leading edge. The support itself may be rigid or elastic in a direction perpendicular to the plate. The plate is excited perpendicular to its plane by a harmonic force at a point near the center of the plate. For all practical purposes the plate can be considered infinitely stiff in spanwise direction. The small-aspect-ratio plate is considered in preference to a two-dimensional plate because of its generality. Two-dimensional flow can, in practice, be obtained by adding fixed walls, between which the plate operates.

Theoretical Aspects

The dynamic analysis of a flexible plate on arbitrary supports, subject to hydrodynamic forces and harmonic excitation, is to a large extent routine. The main problem is the determination of the hydrodynamic pressures. An unconventional aspect in the present case is the computation of the thrust, which, being the product of local pressure and slope, is a second-order effect. Two different approaches can be used, each of which has been given attention in the literature. They will be reviewed briefly.

The first and the most general method employs a kernel-function analysis, which was developed at NACA to determine unsteady aerodynamic pressure distributions[2] but is also used in the analysis of hydrofoils.[3] The method is general in that it is applicable to three-dimensional flow problems and can be extended to include two-dimensional problems and various other conditions that are of practical interest, such as free surface effects, parallel foils, etc. Its main limitation at present, for computational reasons, is the range of applicability of the reduced frequency. This range is adequate in most unsteady aerodynamic problems where the high velocity leads to small reduced frequencies, but this is not so in unsteady hydrodynamics. If this method is to be used in subsequent analyses, it is necessary to improve the accuracy at higher reduced frequencies.

The second, more restricted, method is based on the two-dimensional flow analysis given by Wu[4] and closely paralleled by Siekmann.[5] In both cases the hydrodynamic pressures are determined due to sinusoidal waves, traveling along the plate with arbitrary wave envelope. Their results, which are valid for higher reduced frequencies than the kernel-function method, were substantiated by experiments described by Kelly.[6] The dynamic analysis of the plate is not included in these references. The main concern is the determination of the hydrodynamic pressures due to prescribed waves, leading after integration over the chord to the total thrust, power, and propulsive efficiency.

Some results for prescribed two-dimensional traveling waves were taken from Refs. 4 and 5 and are shown in Fig. 1. The curves were extrapolated for reduced frequencies in excess of 5 from the results given in the references. The values of the thrust coefficient, defined as $c_T = T/[(\pi/2)\ \rho\ U^2wc]$, are shown for a double amplitude at the trailing edge of 12.5% of the chord length. The c_T curves are valid only for the first envelope; the deflection is zero at the leading edge and linearly increasing to a maximum at the trailing edge. Also

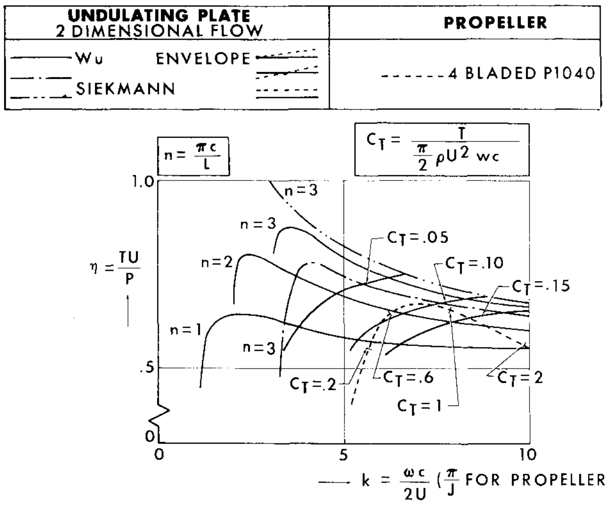

Fig. 1 Propulsion efficiency for various wave envelopes.

given in Fig. 1 is the efficiency of a four-bladed propeller, taken arbitrarily from Ref. 7. The reduced frequency is defined with revolutions instead of cycles and diameter instead of chord length. The number n, used in Fig. 1, is inversely proportional to the wavelength L. The results indicate in general that the efficiency is highly dependent upon the reduced frequency and the wavelength, especially near the maxima. Furthermore, when the efficiency is high the thrust is small, and vice versa.

Although the two methods discussed here both lead to pressure distributions for given displacement (downwash) distributions of the plates, there are some additional factors, different in both methods, that are important in the solution of a dynamic analysis of the plates. Usually in the dynamic analysis of complex structures, the displacements are expressed in terms of orthogonal deflection modes. The equations of motion can then be written in the generalized coordinates corresponding to these modes, and the structural (inertia and stiffness) coefficients can be obtained in a relatively simple manner. The aerodynamic coefficients and the forcing functions corresponding to these modes can also be determined in a routine manner. This is so in the case of the kernel-function method. However, in the traveling-wave analysis a unique transformation from traveling waves into orthogonal modes does not exist. Hence, the application of the second method requires an extension of the usual dynamic analysis, including the selection of appropriate generalized coordinates.

A factor to be considered in the application of the kernel-function method is the leading-edge thrust, which is negligible in aeronautical problems. The leading-edge thrust is a correction, which accounts for a singularity in the pressure distribution and which sometimes represents a sizeable fraction of the total thrust. This can be taken into account as shown in Ref. 4.

Preliminary calculations have reinforced these conclusions. They suggest that the kernel-function method is to be preferred because of its generality and straightforward application. However, its main drawback is the small range of the reduced frequency. If this range cannot be extended substantially, the two-dimensional theory must be used with three-dimensional corrections or applied only to two-dimensional problems.

The results of a study by Bonthron[8] are of interest. He has performed a dynamic analysis of two-dimensional plates, divided chordwise into three rigid sections connected by hinges. The pressures are determined from two-dimensional flow theory and are integrated over the sections. A finite number of equations of motion can be written for these sections. Two types of problems are considered. In one, a phase relation is selected of the motions of the sections so that the resulting motion resembles the swimming of a fish, and the equilibrium conditions are satisfied. In the other type of problem, the leading section is restrained in a horizontal position, and the same phase relations as before are selected for the two rear sections. The efficiencies were computed for some 20 cases, differing in hinge locations and phase relations. The propulsive efficiency appears to be always between 42 and 55% in the first type, whereas in the second type of problem the variation is from 19 to 92%. The conclusion was drawn that there is apparently a lot of rigid-body pitch present when the first section is free. This could be prevented in the other configurations by keeping the first section fixed. Hence, the results show that extremely high efficiencies are possible in particular configurations. These efficiencies are apparently very sensitive to small changes in these configurations. Bonthron's work is of interest for two additional reasons. After studying the fish-propulsion literature and modifying some of the assumptions that have been made, it is concluded that at least for some animals the measured performance is in reasonable agreement with what can be expected. Finally, with respect to the difference in behavior between two- and three-dimensional plates, it is argued that there is probably little difference in propulsive efficiency.

PUP-1 Experiments

Since the literature survey suggested the possibility of high efficient propulsion with undulating plates but did not give immediate clues concerning the design of a system that employs these plates, it was decided to perform a series of relatively simple experiments. The primary objective of these tests remained to determine the potential usefulness of undulating plate propulsion. From the start it was clear that, in order to remain within the bounds of the available resources, the tests would have to be done "in house" using existing equipment.

The PUP-1 (propulsion by undulating plate) watercraft was designed and assembled on short notice using readily available material and equipment (see Fig. 2). Two 150-gal

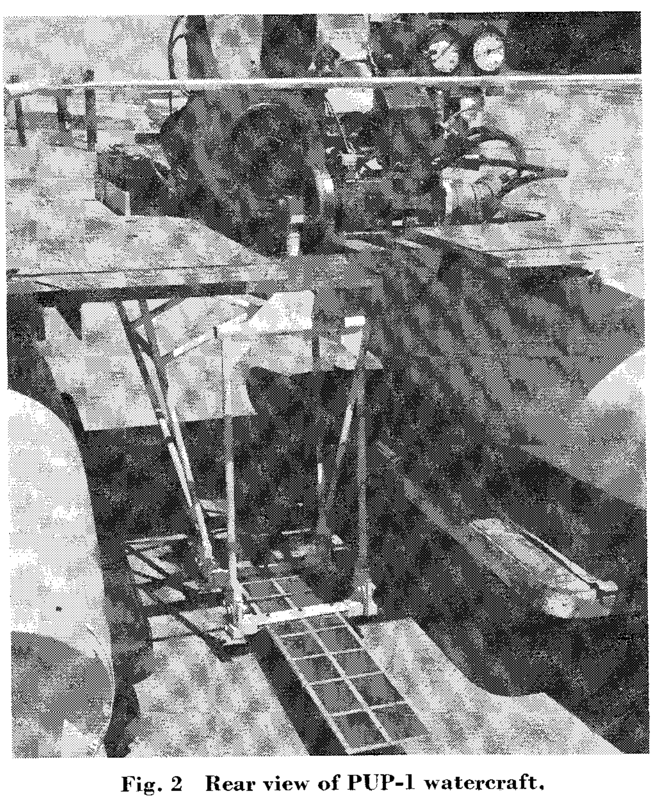

Fig. 2 Rear view of PUP-1 watercraft.

Table 1 Dimensions of plates (in.)

No.	Material	δ	c	N	T_1	δ_1
1	1020	$\frac{3}{16}$	40	1	...	...
2	1020	$\frac{3}{16}$	35	1	...	...
3	1020	$\frac{3}{16}$	30	1	...	...
4	1020	$\frac{1}{8}$	30	1	...	...
5	1020	$\frac{1}{4}$	30	1	...	...
6	1020	$\frac{3}{16}$	34	5	...	...
7	1020	$\frac{3}{16}$	39	10	...	...
8	7075	$\frac{1}{4}$	34	5	...	...
9	7075	$\frac{1}{4}$	34	5	5	$\frac{1}{8}$
10	7075	$\frac{1}{4}$	34	5	8	$\frac{3}{16}$

aircraft drop tanks, acting as floats, are interconnected in catamaran fashion by a steel framework, which serves as a foundation for a 14-hp aircooled gasoline engine and associated equipment. The catamaran arrangement was chosen because it resulted in a relatively stable platform with free access to the water near the center of gravity of the craft. The latter is particularly advantageous in order to avoid excessive pitching of the craft due to the unbalanced vertical force that drives the plate.

The undulating plate is mounted horizontally in the centerline of the watercraft about 2 ft below water level. The plate is hinged at the leading edge in a support frame. The drive link is connected to a hinge located near the center of the plate. This link is driven by an eccentric mechanism, which in turn is driven hydraulically. The hydraulic system was selected because it provides simple clutch and speed control. The relatively large power losses in this system were taken care of easily by choosing an oversize engine.

Control over power output and frequency of excitation is exercised by means of a valve in the hydraulic loop connecting pump and hydraulic fluid tank. The hydraulic motor is connected to this loop in the high-pressure side of the valve. The pressure was monitored with a pressure gage, which allowed accurate readings up to 3000 psi. The hydraulic drive system was calibrated to determine the power output at the eccentric wheel as a function of hydraulic pressure and frequency.

The frequency was measured indirectly by measuring the revolutions per minute of the hydraulic motor with a tachometer. Initially this tachometer was hand held and used only when readings were desired. Later, after one was lost in the pond, the tachometer was mounted permanently, which allowed continuous readings.

The amplitude of the excitation could be selected before each test run by choosing the appropriate hole for the bolt connecting the drive yoke to the eccentric wheel. The amplitudes that were used most often were $\frac{1}{2}$, 1, $1\frac{1}{4}$, and $1\frac{1}{2}$ in.

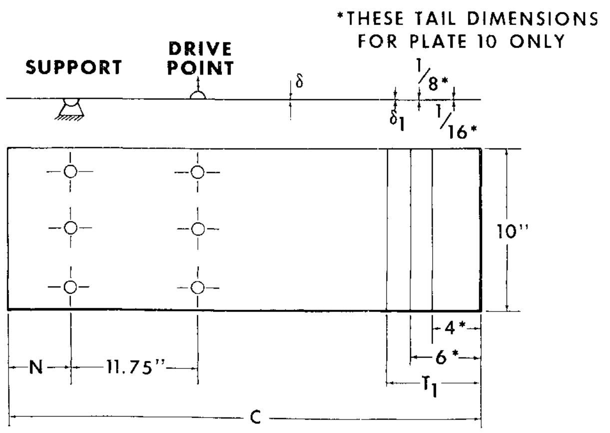

Fig. 3 Dimensions of test specimens.

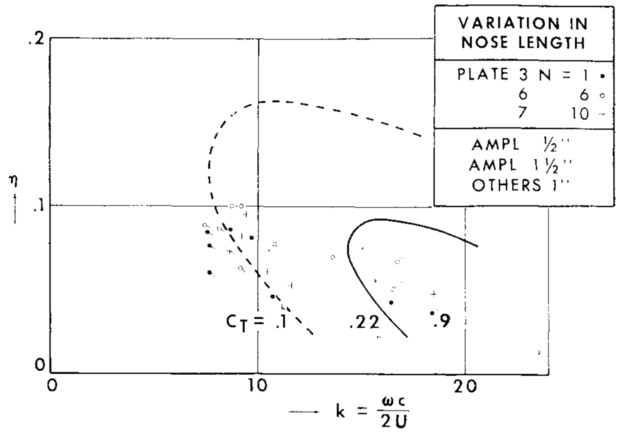

Fig. 4 Efficiencies of plates 3, 6, and 7.

It was decided to measure the speed of the watercraft by making test runs past fixed points along the shore, about 140 ft apart. The elapsed time during each run was measured with a stopwatch. This procedure was followed in all tests. The course was always run in both directions, thus giving a good indication of the influence of the wind.

For the determination of its drag, the watercraft was towed at constant speed along the measured course. The craft was in similar condition as during the tests, with a plate installed but not operating. The towing force was measured with a spring scale. An average parabola was drawn through the test points, resulting in the following relation between thrust (pounds) and speed (feet per second): $T = 1.65\ U^2$.

It is estimated that the thrust values determined from speed measurements will have a range of uncertainty of not more than $\pm 10\%$, especially if the average speed is used of two runs, one with and one against the wind. The latter was done consistently in the reduction of the test data. From the measurements of frequency and hydraulic pressure, the output power to the plate could be determined. The propulsive power followed from the speed and the drag curve. The propulsive efficiency is the ratio of propulsive power to input power.

The test specimens consisted of flat plates with a width of 10 in. and various lengths. They were hinged at or near the leading edge and excited perpendicular to their plane along a line that is $1\frac{3}{4}$ in. aft of the hinge (see Fig. 3).

Ten plates were tested, allowing a preliminary evaluation of the major parameters. Their dimensions are given in Table 1. The major parameters and the corresponding plates, which show the effect of the variation in these parameters, are as follows: chord length = 1, 2, 3; thickness = 3, 4, 5; nose length = 3, 6, 7; material = 6, 8; and tail length = 8, 9, 10.

In principle, the most attractive dimension in one group was carried over to the next group in the sequence shown. The change in material, from 1020 hot-rolled steel plate to aluminum alloy 7075-T651 (75ST), was considered the least interesting because it affects primarily the modulus of elasticity, which plays a role in the plate stiffness term only. Although the modulus of elasticity was lower in plate 8, the over-all stiffness was approximately the same as in plate 6 because of the increase in plate thickness to $\frac{1}{4}$ in. An additional reason for the change to 75ST was the necessity of machining the tail sections of plates 9 and 10 which could be done more easily with 75ST.

Test Results

Measurements were taken at a minimum of three different frequencies and three different amplitudes for each of the 10 plates, in an attempt to cover a range of the parameters. The maximum velocities attained in the tests are given in Table 2.

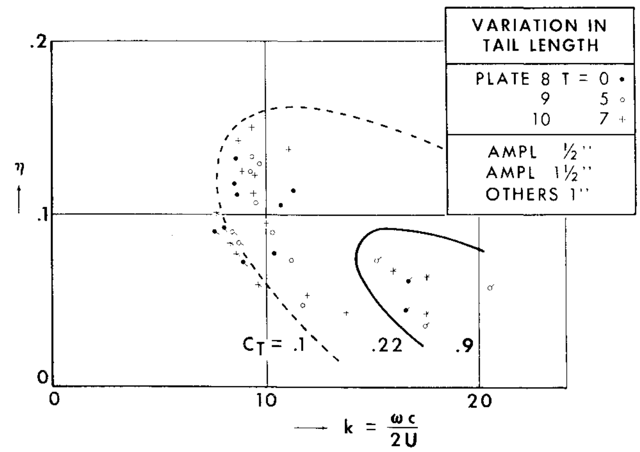

Fig. 5 Efficiencies of plates 8, 9, and 10.

The efficiencies are plotted against the reduced frequencies for plates 3, 6, and 7 in Fig. 4 and plates 8, 9, and 10 in Fig. 5. The results for the other plates are similar to those shown in Fig. 4 but are less interesting because they cover a smaller range of parameters.

Three factors should be kept in mind concerning these results. In the first place, the use of the watercraft, with its fixed relation between the thrust and the velocity, limits the tests to a constant value of c_T. In the second place, the hydraulic motor limits the maximum power output as a function of the frequency. In the third place, the power and the thrust are proportional to the square of the deflection amplitude. The combined effect of these factors can be characterized as one of matching the plate to the watercraft. The frequency and deflection amplitude are always such that the available power equals the power that is required to generate in the plate at the particular velocity a thrust which is sufficient to overcome the drag of the watercraft at that velocity. This matching shows up in Table 2. By retaining in subsequent specimens the improvements found in previous tests, the later plates were better matched to the watercraft, and more power could be applied, resulting in higher speeds. Nevertheless, the design improvements usually also produced improvements in efficiency, but these were of less consequence in the performance of the plate than the increase of the available power.

In order to separate the influence of the deflection amplitude from the influence of the other variables, a correction was applied to the values of c_T in Figs. 4 and 5. In principle, the same value of c_T applies to all test points, of the order of 0.22 and varying slightly with the plate area. Since the thrust and the power are proportional to the square of the deflection amplitude, provided that the frequency and velocity remain constant, the thrust coefficient is also proportional to the square of the deflection amplitude, whereas the efficiency and reduced frequency are not affected. The values of c_T indicated in Figs. 4 and 5 are the thrust coefficients corrected for a deflection amplitude of 1 in. at the drive point. The curves shown in Figs. 4 and 5 separate regions with different c_T but with the same deflection amplitude or, if the correction is ignored, regions with the same c_T but with different amplitudes. The same curves were drawn in both figures.

The test points in Fig. 5 for plate 10 at 1-in. amplitude form a good illustration of the behavior of the plates. The sequence of measurements during the test represents a gradual increase in power. For low values of the frequency the efficiency increases with decreasing k, or the velocity increases faster than the frequency. At about 5.2 cps, a minimum value of k is reached where frequency and velocity change equally fast. Above this frequency the efficiency increases with increasing k, or, in other words, the frequency increases faster than the velocity. At still higher power levels the frequency increases much faster, resulting in a decrease of efficiency with a sharply increasing k.

The test points at frequencies and velocities higher than those corresponding to the condition of maximum efficiency proved difficult to obtain. In some instances the frequencies increased suddenly in this range, giving the impression of a jump to a much higher value of k due to a breakdown of circulation. It appears that the accurate measurement of the efficiency curve near the points of minimum k and maximum efficiency requires a better control over the input power, frequency, and velocity.

The influence of the major parameters within each group of plates is difficult to assess. The criterion during the tests for best performance was the maximum velocity. However, since the maximum power used in the first few plates was less than available, higher velocities could have been attained at larger amplitudes. Nevertheless, judging from Figs. 4 and 5, it appears that the medium nose length (plate 6) is superior to the short and long nose lengths (plates 3 and 7) and that the most flexible tail performed better than the less flexible.

The influence of the chord length, the thickness, and the material is not clearly determined in these tests. It is felt that the influence of chord length is mainly one of scale of the dimensions, whereas the influence of the last two is exclusively on the stiffness of the plates. Both influences affect of course the maximum velocity and efficiency, but it is considered that the number of test conditions is not sufficient to come to more specific conclusions than the trend indicated in Table 2.

The wave parameter n could not be determined in the present tests. A few underwater movie shots taken in a small shallow pool, too small to exercise control over thrust and velocity, indicate that, at least in plate 10, values of n of the order of 1 were obtained.

Measurements of acoustic energy in the water would have been of interest. It seemed, however, wiser to postpone these until a plate with higher efficiency was available.

Two of the plates failed during the tests because of fatigue. Both times the failures occurred in steel specimens near the drive link attachments. Inspection revealed a stress concentration at a sharp edge. After removing this stress concentration, no more failures occurred. However, because of these failures, the amplitudes of the first few plates were not increased above $1\frac{1}{2}$ in., although there was sufficient power available. A few additional runs were made with one of the plates attached but not operating and the watercraft propelled by a 4.5-hp Champion outboard motor. In this condition a speed of about 5.8 fps was reached at maximum power. This should be compared with the maximum velocity attained in the tests with plate 10 of 5.1 fps.

Discussion

Because of the preliminary nature of the tests, the results are insufficient to establish clearly the influence of the design parameters. What could be determined were the values of

Table 2 Measurements at maximum velocity

No.	Amplitude, in.	f, cps	U_{max}, fps	T, lb	P, hp
1	1.5	4.75	2.7	12	1.2
2	1.5	4.28	2.6	11.5	1.1
3	1.5	3.47	3.4	19	1.4
4	1.5	3.05	3.1	16	0.7
5	1.5	5.17	3.35	18.5	2.0
6	1.0	4.52	3.7	23	1.4
7	1.0	5.24	3.7	23	2.0
8	1.0	6.38	4.5	34	2.6
9	1.0	6.0	4.75	37	2.5
10	1.0	7.5	5.1	43	2.85

k, c_T, and η that can be expected when using simple, flat, small-aspect-ratio plates. The values of k and c_T are more significant in this respect than η, because the latter merely affects the amount of input power, which is not always in short supply.

The values of k are, in general, larger than expected. This can be interpreted as a requirement for relatively high frequencies (high for transverse vibration of flat plates, that is) or as a conclusion that the velocity will always be relatively low. Higher velocities can be reached only with stiffer plates. Higher operating frequencies are also an advantage in the oscillating drive system, because they permit higher power levels at the same deflection.

The values of c_T depend to a large extent on the deflection amplitude and are, therefore, limited by the maximum allowable load in the plate. This load will probably be determined by fatigue considerations. A good appreciation of the limits on c_T can be obtained only from detailed analysis or more accurate experiments, including the stress distribution in the plate. Although fatigue may be the limiting factor in the design, it should be noted that this need not be too severe in practice, since in a constant-frequency system the number of cycles can be monitored, and the plates can be replaced easily.

The small-aspect-ratio plates used in the tests were exposed to an essentially three-dimensional flow. The difference between the results in this type of flow and those in the two-dimensional cases of the theoretical analyses is not known. However, upon reflection, it seems reasonable to consider one major feature of small-aspect-ratio plates to be the dependence of the virtual mass of the fluid on the local width rather than the chord. Then, the distribution of the width over the chord length becomes an additional means of influencing the waveform, more important than the mass of the plate itself because that can be varied only within a small range.

A relative increase of the deflection and decrease of the wavelength toward the trailing edge implies a large mass (width) and stiffness of the forward part and a small stiffness and width of the rear. However, the thrust is primarily generated by the tail as a product of pressure, slope, and tail area. Hence, it is necessary to compromise there. If the span of the tail is too large, the deflection and the slope will be too small, and if the span is too small, the area will be too small. It can be concluded that a small-aspect-ratio undulating plate used for propulsion should have a "fish" shape.

In the two-dimensional case of large-aspect-ratio plates, the freedom to select the most favorable width distribution is absent, and the mode shape can be influenced only by the thickness distribution. Furthermore, the important influence of the tailspan on the thrust is completely lost. Since the deflections will be, in general, smaller than in the three-dimensional case for the same pressures, it seems necessary to allow larger tail deflections relative to the rest of the plate than in the three-dimensional case for the same thrust. If this is so, equal deflections would produce a larger thrust in the two-dimensional than in the three-dimensional case. It is not obvious how this would affect the efficiency. There is some doubt that two-dimensional flow, which may result in higher pressures, will necessarily lead to higher efficiency.

A compromise may be reached between the two- and three-dimensional cases with a flat plate between two walls. These walls form a duct with a width that can be assumed, in general, to be variable. This appears to be a more promising case, because the higher pressures of an essentially two-dimensional flow are combined with the freedom to choose the most favorable width distribution.

Another effect, not discussed so far, is that of a fixed wall parallel to the plate. An equivalent effect would be due to two parallel plates operating exactly out of phase. This arrangement has a practical advantage in that the transverse loads on the plates and the supports are also out of phase and, therefore, cancel each other. Hence, a propulsion unit can

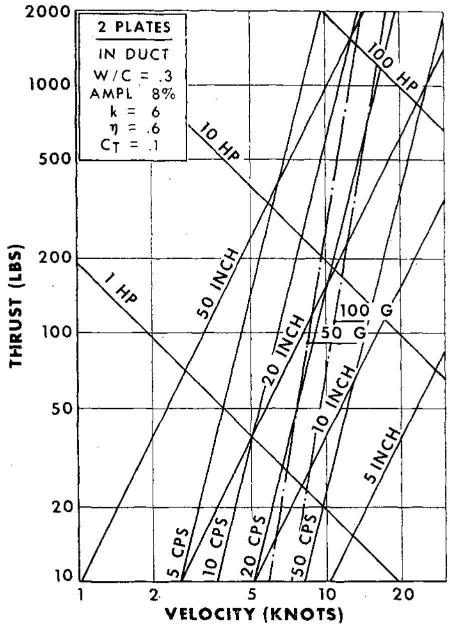

Fig. 6 Design chart for one combination of parameters.

be designed, consisting of an even number of parallel plates, each of which operates out of phase with respect to its neighbors, so that all internal loads except the thrust forces are canceled. The basic element of this propulsion unit consists of two parallel plates operating out of phase. Although there is at present no experimental evidence supporting this, it is believed that in this basic element the thrust will be substantially better than in the individual plates separately. Also, the efficiency may be improved greatly.

Applications

On the basis of the limited information obtained so far, a prediction must be made concerning possible applications. Two specific designs have been selected which are considered to show the most promise. Each consists of an even number of undulating plates, excited by an oscillating drive system. In both cases this drive system deforms elastically and exhibits at least one stationary point where it is supported and sealed in the hull. In one case the oscillating system is an elastic beam vibrating in transverse direction, and in the other case it is an elastic shaft oscillating in torsion. Finally, an essential element of the propulsion systems is the mechanical oscillator, because its use results in some unique features of the systems.

The first design employs two small-aspect-ratio plates parallel to each other and driven out of phase. It seems not unreasonable to assume that the value of the parameters that can be reached by this combination is $\eta = 0.6$, $k = 6$, and $c_T = 0.1$. The relation between the plate dimensions and the performance under those conditions is shown in Fig. 6. The thrust and power are given for the pair of plates. The remark "in duct" means that the set of parameters is probably valid only if the plates operate between two parallel walls. This is, however, not known and may not be necessary. The figure corresponds to only one set of parameters: the one that is supposed to give optimum performance. It does not show how the plates arrived in that condition or what the performance at frequency lower than the optimum will be. The amplitude of the wave on which Fig. 6 is based is 8% of the chord at the trailing edge, a value that is considered reasonable if not low. The acceleration of the trailing edge is also indicated. This gives a reasonable impression of the practical limits of designs set by high loads on the plates.

It appears that the performance is limited to a range of rather low velocities for the combination of parameters chosen. The thrust is reasonable, and practically any requirement can be met by employing parallel sets of plates. Higher velocities are possible only at higher frequencies and with stiffer plates. These allow only small deflections and will operate at lower c_T. Therefore, it seems likely that higher velocities will be obtained only with systems consisting of a larger number of parallel plates that are relatively small and stiff.

This leads to the second design that is considered to show promise. It consists of an even number of smaller plates, arranged in axially symmetric fashion on the outside of a cylinder, either within or without a shroud. Although the plates are supported by hinges attached to the cylinder, they are driven by a common ring attached to a drive system oscillating in torsion. In a more advanced version, consecutive plates would be driven alternately by two rings attached to two concentric drive systems oscillating out of phase.

Both designs benefit from the use of mechanical oscillators in the drive system. Since undulating plates will operate best within a narrow range of frequencies, the mechanical oscillators can be designed easily for maximum power and deflection at that frequency. Furthermore, the output power at a given frequency depends directly on the deflection. This is very useful for the control of the power. In a system in which two oscillators operate out of phase, as in the first design, the relative deflections of the oscillator houses can be controlled simply. In practice, the oscillators can be driven at design speed without transmitting power by locking the houses together.

In this condition, starting and running up to speed will be done in an extremely short time. At the operating frequency the houses are gradually unlocked, allowing a build-up of deflection. At maximum power the houses are completely free. It may prove to be an attractive feature of this design that it can be operated in bursts with the oscillators running at constant speed, simply by locking and unlocking the oscillator houses. The system is also hydrodynamically well suited for this operation in bursts because it has, in contrast with ordinary propellers, low drag in unpowered glide motion.

Conclusions

It is apparent that the application of an underwater propulsion system requires strong justification, since it entails the exchange of the conventional propeller for a complex system consisting of mechanical oscillators, vibrating beams or shafts, and a number of undulating plates. The system is also limited to medium velocities and, therefore, practically out of the range of high-performance vehicles. Nevertheless, the complexity may be the price worth paying for the solution of the problems occurring in a few specific areas where high performance is not a prerequisite.

One of those areas is that of the very deeply submerged vehicle. The difficulty of sealing rotating shafts may be solved by applying the nonrotating seals of the oscillating system. Another is a very shallow, muddy environment through which the oscillating plate will glide like a knife. Similarly, undulating plate propulsion may be the only possible form of underwater propulsion through waters infested with weeds. Other reasons for discarding propellers in favor of undulating plates are the danger of exposed blades to underwater swimmers or the noise generated by the blades. It has not been demonstrated yet that undulating plates are relatively quiet, but since the plates vibrate at a rather constant subaudio frequency it is expected that they will radiate mainly at this and practically not at higher frequencies.

The present paper deals mainly with the prospects of the concept of propulsion by undulating plates. The literature study and the PUP-1 experiments have shown the feasibility of the concept. For a more detailed evaluation, it is necessary to acquire a theoretical method for the analysis of the thrust and efficiency of flat plates on various supports and under arbitrary boundary conditions. The capabilities of this method must be substantiated by the results of an experimental program, preferably carried out in a towing tank.

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